TUYMAADA juniors - 2005
Day 1

1. In each cell of the table 3×3 there is one of the numbers $1, 2$ and 3. Dima counted the sum of the numbers in each row and in each column. What is the greatest number of different sums he could get? (S. Volchenkov)

2. Points X and Y are the midpoints of the sides AB and AC of the triangle ABC, I is the center of its inscribed circle, K is the point of tangency of the inscribed circles with side BC. The external angle bisector at the vertex B intersects the line XY at the point P, and the external angle bisector at the vertex of C intersects XY at Q. Prove that the area of the quadrilateral $PKQI$ is equal to half the area of the triangle ABC. (S. Berlov)

3. Tram ticket costs 1 Tug. 20 passengers have only coins in denominations of 2 and 5 Tug, while the conductor has nothing at all. It turned out that all passengers were able to pay the fare and get change. What is the smallest total amount of Tug the passengers might have?

4. The organizers of a mathematical congress found that if they accommodate any participant in a room the rest can be accommodated in double rooms so that 2 persons living in each room know each other. Prove that every participant can organize a round table on graph theory for himself and an even number of other people so that each participant of the round table knows both his neighbors.(S. Berlov, S. Ivanov)

(problem 3 from the seniors, solutions can be found on AoPS.)

Day 2

5. Given the quadratic trinomial $f(x) = x^2 + ax + b$ with integer coefficients, satisfying the inequality $f(x) \geq -\frac{9}{10}$ for any x. Prove that $f(x) \geq -\frac{1}{4}$ for any x. (A. Khrabrov)

(solutions can be found on AoPS)

6. Along the direct highway Tmutarakan - Uryupinsk at points A_1, A_2, $\ldots$, A_{100} are the towers of the DPS mobile operator, and in points B_1, B_2, $\ldots$, B_{100} are the towers of the "Horn" company. (Tower numbering may not coincide with the order of their location along the highway.) Each tower operates at a distance of 10 km in both directions along the highway. It is known that $A_iA_k \geq B_iB_k$ for any i, $k \leq 100$. Prove that the total length of all sections of the highway covered by the DPS network is not less than the length of the sections covered by the Horn network.

7. The point I is the center of the inscribed circle of the triangle ABC. The points B_1 and C_1 are the midpoints of the sides AC and AB, respectively. It is known that $\angle BIC_1 + \angle CIB_1 = 180^\circ$. Prove the equality $AB + AC = 3BC$. (F. Bakharev, D. Rostovsky)

(solutions can be found on AoPS)

8. A sequence of natural numbers is constructed according to the following rule: each term, starting with the second one, is obtained from the previous one by adding the product of its prime divisors (for example, after the number 12 comes the number 18, and after the number 125, the number 130). Prove that any two sequences constructed in this way have a common term. (A. Golovanov, incorrect translation from Mongolian)